IDENTIFYING THE THERMOPHYSICAL PROPERTIES OF SUBSTANCES

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UDC 536.2.01

Schemes (procedures) are presented for identifying thermophysical properties. The schemes are constructed on the basis of the structural analysis of problems of heat conductivity.

Identification is the construction of mathematical models on the basis of observations of phenomena or processes occuring in nature or industry [1-5]. Methods have been developed in identification theory for constructing mathematical models of dynamic systems from the results of measurements having errors. Thus, identification includes: 1) planning of the experiment; 2) determination of the class to which the unknown operator belongs; 3) estimation of unknown parameters of the system operator.

In those cases where the structure of the operator (mathematical model) is known from a priori information, identification involves only steps 1 and 3. The mathematical model of a linear system may be represented either by a weight function or a transfer function; the transition can always be made from these functions to a differential operator.*

The mathematical model of a physical process is not and cannot be adequate to the process itself. Thus, the existence of a solution to a physical problem does not always imply the existence of a solution for its corresponding mathematical problem: the existence of the latter can be proven or disproved only through mathematical investigation. Thus, in studying a mathematical model, the physical considerations are necessarily of a tentative and probable nature, and by themselves cannot serve as proof of the adequacy of the model.

The construction of schemes (procedures) of identification is based on the following arguments. Let a dynamic system of any type be under the influence of an effect characterized by vector u of the "input" variables (determined or arbitrary). Then the dynamic system, in accordance with its internal properties, transforms these input variables u into output variables y, which may be represented as

$$y = Du\,,$$

which should be interpreted as follows: a certain physical operator D acts on the input variable u in such a way that it is transformed into output variable y. The physical operator D may, with a certain degree of accuracy, be represented by mathematical operator K, so that the real mapping of the input variables u and output variables y will correspond to its mathematical model:

$$y_{\mathrm{m}} = Ku\,.$$

In this notation, it is assumed that u is measured exactly and is transformed approximately into the quantity y_m; thus, the subscript m is used with y. Finding operator K is the procedure of identification. The structure of operator K and the value of its coefficients should in a certain sense be optimum. The values of the parameters of the object (thermophysical properties, for example) cannot be obtained by direct observation, but the output signal is related to their numerical values. Thus, the criterion for an optimum determination of operator K should be a functional of the output variables of the object

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*It is assumed that the coefficients of the differential equations describing the process are independent of time. Otherwise, such a transition from a weight or transfer function to a differential operator is difficult.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 3, pp. 416-427, September, 1980. Original article submitted May 16, 1980.

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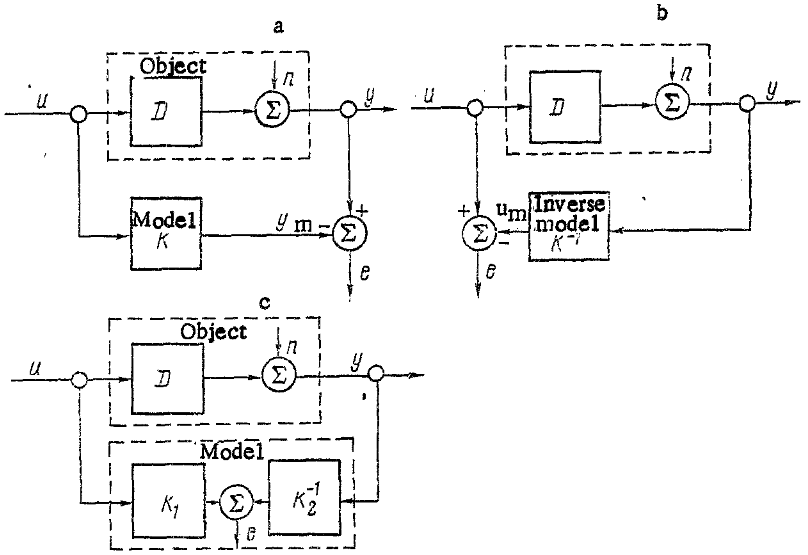

Fig. 1. Identification schemes: D) physical operator of the object; K) mathematical operator.

of the study, its model, and the estimates* of the parameters of the object. We should note that the error

$$e = y(t, b, n) - y_m(t, \beta, 0), \ (y_m = Ku)$$

is a function of the values of y and y_m at moment of time t. Here, it is assumed that the model is adequate and that the measurements are made with an error. Thus, the error e in this case is due to the error in the measurement of y and the estimate of the value of parameter b, assumed to be equal to β.

It is assumed that the object and its model are similar in structure (the measurement error does not alter the form of the model). This means that the output of the object

$$y = f(u, b, n)$$

and the output of the model

$$y_m = f(u, \beta, 0),$$

where b is a parameter of the object; β, an estimate of parameter b; n(t), measurement error (described by equations identical in structure).

If the analytic expression is known for operator K, which depends on constant parameters b, and measurements of the output variable y and input variable u are available, an estimate of b may be found from the condition of proximity of the object and its model, defined by the minimization of the functional:

$$E\{y, y_m, \beta\} = \int_{t-T}^{t} \{y(t, b, n) - y_m(t, \beta, 0)\}^2 dt,$$

where T is the duration of the measurement.

The error will be minimal when the value of β_i is determined from the system of (m + 1) equations with (m + 1) unknown estimates (β_0, β_1, ..., β_m):

$$\frac{\partial E}{\partial \beta_i} = 0, \ i = 0, 1, 2, \ldots, m.$$

In the general case, when y_m depends nonlinearly on b, this functional may have several extremes.

Identification schemes are depicted in Fig. 1a and b; in Fig. 1a, when a known effect is acting on the object and the model and the output signals of the object and model,

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*An estimate here means numerical values to which the rule of estimation is applied in a specific case (an experimentally determined value of the parameter).

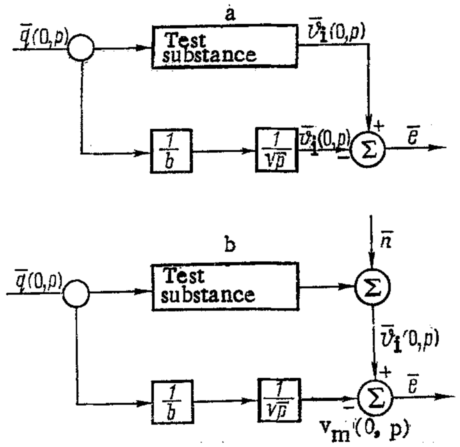

Fig. 2. Schemes of identification (determination) of the coefficient of thermal activity.

distorted by the error n(t), are compared; in Fig. 1b, when the effect acting on the object is unknown, and the output variable of the object is measured with an error.

The notation K^{-1} (inverse operator) assumes that the model is reversible — a unique input signal may always be found for a given input. In certain cases, the identification scheme may be as shown in Fig. 1c. The usefulness of this scheme is that the output variable (its L-transform) is often represented as follows:

$$y = K_1K_2u\,.$$

Usually, either it is difficult to find the inverse operator $[K_1K_2]^{-1}$ or the expression found for it cannot be used. In many cases, these difficulties can be avoided by representing $y = K_1K_2u$ in the form

$$K_2^{-1}y = K_1u\,.$$

Operator K_1 can always be found, however, so the problem becomes simpler — we have to find inverse operator K_2^{-1}, the effect of which on a certain value of y is such that it is transformed into the already known quantity K_1u. Further construction of identification schemes will proceed in the region of the L-transforms on the basis of structural schemes (models) of problems of thermal conductivity [6]. It should be pointed out that the time region of the error e is a function of the measured values, and only the criterion of the estimate of the proximity of the model to the object is a functional, the value of which depends on the observation time.

In the region of the L-transforms, the error $\bar{e} = \bar{y}(p, b, n) - \bar{y}_m(p, \beta, 0)$ is already a functional. Here, the fact that the L-transform of y_m is exact and that u is an inexact L-transform of measurements of y is important. This inexactness is due to the following: a) the finite observation (measurement) time; b) measurement error y; c) the error in computing $\bar{y}$.

For the error due to the finite measurement time to be negligibly small, it should be no larger than the errors in the measurement of y and computation of $\bar{y}$. To this end, it is necessary to select the proper value for pt_p. The estimate of b in the L-transform region is determined from the condition of minimization of the functional

$$E\{\bar{y}, \bar{y}_m, \beta\} = [\bar{y}(p, b, n) - \bar{y}_m(p, \beta, 0)]^2.$$

In developing methods of determining thermophysical properties, it is usually assumed that the properties of the substance do not change with time and are independent of temperature. Such assumptions are sufficient for obtaining an analytical relationship between the observed quantities. Further, if such a relationship is found, the solution obtained is written relative to the sought coefficient.

To compute the value of a given constant coefficient, it is sufficient to obtain values of the measured quantities (temperatures) for a single moment of time. However, the relationship between these quantities (temperatures) before the measurement, when they are unknown, may be expressed by a convolution integral with a corresponding kernel. In this case, we need to record thermograms and approximate them with convenient expressions, as is typical of cases where the thermophysical properties of the object are measured during its normal operation.

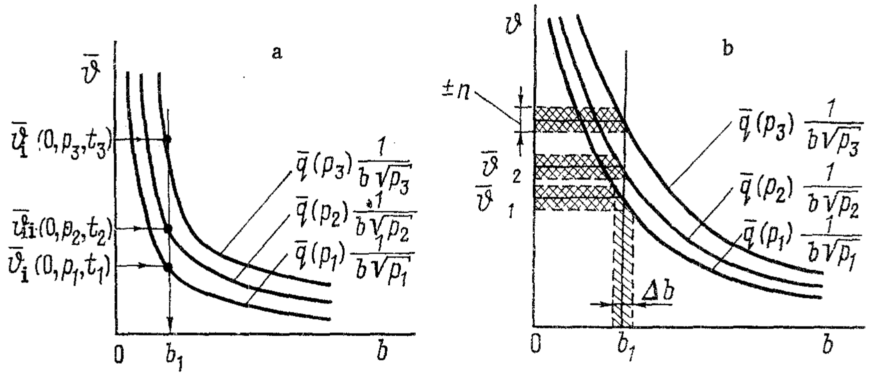

Fig. 3. Graphical determination of the coefficient of thermal activity b when the temperature is measured exactly (a) and with an error (b).

We will show how the structural scheme in [6] can be used to obtain the fundamental relations between the measured and sought quantities, which can in turn be used to develop methods and procedures for determining the thermophysical properties of the test medium. To this end, let us examine several examples.

Example. We need to determine (estimate) the value of the coefficient of thermal activity b from measurements of the temperature $\vartheta(0, t)$ of the surface of a half space ($x = 0$) and the heat flux incident on the surface.

The scheme of identification (determination) of the coefficient of thermal activity is shown in Fig. 2. The mathematical model of the half space (or semiinfinite rod, the lateral surface of which is thermally insulated) is represented by the operator $1/b\sqrt{p}$. Depending on the program of the experiment, the dependent and independent variables should be indicated, i.e., the variables that are assigned (the input variable and the law of its change) and that are to be measured (the output variable) should be indicated.

The schemes in Fig. 2a and b were constructed on the assumption that the heat flux is assigned (known exactly) and that the temperature $\vartheta(0, t)$ is measured exactly (Fig. 2a). The error e is determined by the estimate of coefficient b, which may be found from the condition of the minimum of the standard deviation:

$$E\{\bar{\vartheta}_i, \vartheta_m, b\} = [\bar{\vartheta}_i(0, p) - \frac{1}{b\sqrt{p}}\bar{q}(0, p)]^2 = \bar{e}^2,$$

i.e., b is found from the expression

$$\frac{de(b)}{d} = 2[\bar{\vartheta}_i(0, p) - \frac{1}{b\sqrt{p}}\bar{q}(0, p)]\left(\frac{1}{b^2\sqrt{p}}\right)\bar{q}(0, p) = 0,$$

from which

$$b = \frac{1}{\sqrt{p}}\frac{\bar{q}(0, p)}{\bar{\vartheta}_i(0, p)}.$$

In the general case, the Laplace transform parameter p is a complex number. In particular, it may be a positive real number. Then the value of the quantity $\bar{q}(0, p)$ may be regarded as the result of the effect (integral) of the heat flux $q(0, p)$ over an infinitely large interval of time with weight function e^{-pt}. The quantity $\bar{\vartheta}_i(0, p)$ may also be regarded as the result of the effect of the temperature $\vartheta(0, t)$, but for a finite interval of time — measurement time t_i with weight function e^{-pt}. The latter is very important, since it means that the difference

$$\int_0^{t_m}\vartheta_i(0, t)e^{-pt}dt - \bar{q}(0, p)\frac{1}{b\sqrt{p}}$$

will be small only at certain values of the number p, satisfying a certain criterion dependent on measurement time t_i.

The existence of such p, at which the difference

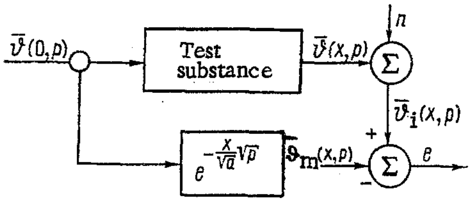

Fig. 4

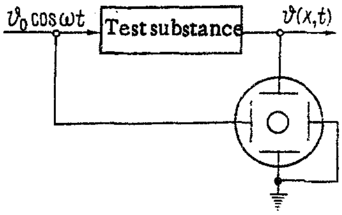

Fig. 5

Fig. 4. Scheme of identification of thermal conductivity.

Fig. 5. Scheme for determining thermal conductivity from Lissajous figures.

$$\int_0^t \vartheta_i(0,\ t)\,e^{-pt}dt-\bar q(0,\ p)\frac{1}{b\sqrt{p}}\leqslant e,$$

where e is small relative to the assigned number, may be regarded as a criterion of adequacy of the linear model throughout the measurement time interval.

The value of e characterizes the degree of proximity of the model to the object. In particular, if we assume that the measurement of the temperature $\vartheta_i(0,\ t)$ and the computation of its L-transform $\bar\vartheta_i(0,\ p)$ are performed exactly, then the difference

$$\bar\vartheta_i(0,\ p)-\bar q(p)\frac{1}{b\sqrt{p}}=0,$$

when the value of b is found from this equation. A graphical solution to the equation is shown in Fig. 3a. However, in reality, measurements of $\vartheta_i(0,\ t)$ are always performed with an error

$$\vartheta_{\mathrm{tru}}(0,\ t)=\vartheta_i(0,\ t)\pm n,$$

where n is the measurement error; L, transform of the quantity ϑ and contains an additional error attributable to the finiteness of the measurement time t_i. Consequently, the value of b should be estimated from the condition of minimization of the functional

$$\left[\bar\vartheta_i\pm\bar n-\frac{1}{b\sqrt{p}}\bar q(0,\ p)\right]^2,$$

which leads to the expression

$$\bar\vartheta_i\pm\bar n=\frac{1}{b\sqrt{p}}\bar q(0,\ p),$$

from which

$$b=\frac{1}{b\sqrt{p}}\ \frac{\bar q(p)}{\bar\vartheta_i(p)\pm n}.$$

The graphical solution of the equation

$$\bar\vartheta_i\pm n=\frac{1}{b\sqrt{p}}\bar q(0,\ p)$$

is shown in Fig. 3b.

We may take as the estimated value of b any value from the interval Δb, determined by the error in the measurement of the temperature ϑ_i, the computing error, and the adequacy of the mathematical model. If Δb is too large, it may be assumed that the model — particularly in the case of linear models — is inadequate.

Example. We need to determine (estimate) the value of thermal conductivity a from measurements of the temperature $\vartheta(0,\ t)$ of the surface of a half space $(x=0)$ and the temperature $\vartheta(x,\ t)$ a certain distance x from the surface.

The scheme of identification (determination) of thermal conductivity is shown in Fig. 4. The model of the half space is represented by the operator $e(-x/\sqrt{a}\sqrt{p}$. The temperature $\vartheta(0,\ t)$ is measured exactly, while the temperature $\vartheta(x,\ t)$ is measured with a certain error. The value of a may be found from the condition of the minimum of the standard deviation

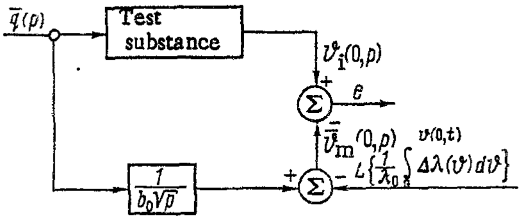

Fig. 6. Scheme for identifying the temperature dependence of thermal conductivity.

$$[\bar{\vartheta}_i(x,\ p)-e^{-\frac{x}{\sqrt{a}}\sqrt{p}}\bar{\vartheta}(0,\ p)]^2=e^2,$$

i.e., the value of a is found from the expression

$$\frac{de(a)}{da}=2[\bar{\vartheta}_i(x,\ p)-e^{-\frac{x}{\sqrt{a}}\sqrt{p}}\bar{\vartheta}(0,\ p)]\cdot 2x\sqrt{ap}\,e^{-\frac{x}{\sqrt{a}}\sqrt{p}}\bar{\vartheta}(0,\ p)=0$$

and has the form

$$a=\frac{px^2}{\ln^2[\bar{\vartheta}_i(x,\ p)/\bar{\vartheta}(0,\ p)]}.^*$$

In computing a, parameter p may be assigned several values. Then

$$a=\sum_{i=1}^{n}\frac{p_i x^2}{\ln^2[\bar{\vartheta}_i(x,\ p_i)/\bar{\vartheta}(0,p_i)]}.$$

If the temperature inside the rod (half space) is measured between points located distances x_1 and x_2 from the surface, the theoretical formula has the form

$$a=\sum_{i=1}^{n} g_i p_i \Delta x^2/\ln^2[\bar{\vartheta}(x_2,\ p)/\bar{\vartheta}(x_1,\ p_i)],$$

$$\Delta x=x_1-x_2,\quad \bar{\vartheta}(x_2,\ p)<\bar{\vartheta}(x_1,\ p).$$

In a number of cases, it is particularly convenient to obtain data on the thermal conductivity of a substance using frequency methods, when heating conditions change harmonically over time. The mathematical formulation of a given heat transfer problem in a substance depends substantially on the mode of the harmonic action, which is taken into account either by means of boundary conditions or by the introduction into the equation of a term which describes the effect of internal heat sources. In the first case, a harmonic effect may be realized by the following methods: the passage of a sinusoidal electrical current through a thin conducting film deposited on the surface of the test substance; through the action of a modulated laser beam on the surface of the test substance. In the second case, a harmonic effect can be obtained by the passage of a sinusoidal electrical current through the test substance, which performs the function of a thermistor. It is understood that a specimen of the appropriate shape and dimensions is to be prepared from this substance.

Example. We need to determine the thermal conductivity a from measurements of the temperature $\vartheta(0,\ t)$ of the surface of the end of a semiinfinite rod thermally insulated on its lateral surface and the temperature $\vartheta(x,\ t)$ a certain distance x from the end surface. It is assumed that $\vartheta(0,\ t)=\vartheta_0\cos\omega t$. Heat transfer is described by the standard equation of thermal conductivity. Then

$$\vartheta(x,\ t)=\mathrm{Re}\,\vartheta_0 e^{j\omega t}G(j\omega),$$

where Re is the real part of the expression

$$\vartheta_0 e^{j\omega t}G(j\omega);\quad G(j\omega)=e^{-\frac{x}{\sqrt{2a}}\sqrt{\omega}}\,e^{-j\frac{x}{\sqrt{2a}}\sqrt{\omega}}.$$

Having isolated the real part, we obtain

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*Measurements analyzed using this formula are shown in [9] for asphalt.

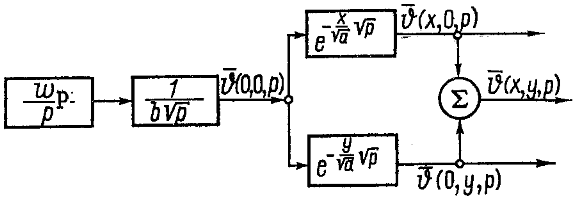

Fig. 7. Structural scheme of two-dimensional problem for thermal conductivity.

$$\vartheta(x,\ t)=\vartheta_0 e^{-\frac{x}{\sqrt{2a}}\sqrt{\omega}}\cos\left(\omega t-\frac{x}{\sqrt{2a}}\sqrt{\omega}\right).$$

To compute the thermal conductivity a, we should measure the temperature $\vartheta(0,\ t)$ and the amplitude of the temperature $\vartheta(x,\ t)$ in cross section x at a fixed frequency. We can measure the phase angle $\varphi=\frac{x}{\sqrt{2a}}\sqrt{\omega}$ and then compute the value of a

$$a=\frac{x^2}{2\varphi^2}\omega=\frac{x^2}{\varphi^2}\pi f=\frac{x^2}{\varphi^2}\frac{\pi}{T},$$

where $\omega=2\pi f$; $f=1/T$; T is the period of temperature measurement.

The data needed for the calculations are taken from thermograms of $\vartheta(0,\ t)$ and $\vartheta(x,\ t)$. The thermal conductivity can also be determined in another way (Fig. 5).

A voltage proportional to the temperature $\vartheta(0,\ t)$ is applied to the vertical plates of an oscillograph, while a voltage proportional to the temperature $\vartheta(x,\ t)$ is applied to the horizontal plates. A figure formed by an orthogonal combination of two harmonically changing functions will then be formed on the oscillograph screen:

$$\frac{\vartheta(0,\ t)}{\vartheta_0}=\cos\omega t,$$

$$\frac{\vartheta(x,\ t)}{\vartheta_0 e^{-\frac{x}{\sqrt{2a}}\sqrt{\omega}}}=\cos(\omega t-\varphi).$$

Having excluded time t from this system of equations, we obtain the expression

$$\frac{\vartheta^2(x,\ t)}{\vartheta_0^2 e^{-2\frac{x}{\sqrt{2a}}\sqrt{\omega}}}+\frac{\vartheta^2(0,\ t)}{\vartheta_0^2}-2\frac{\vartheta(0,\ t)}{\vartheta_0}\frac{\vartheta(x,\ t)}{\vartheta_0 e^{-\frac{x}{\sqrt{2a}}\sqrt{\omega}}}\cos\varphi=\sin^2\varphi,$$

which is the equation of an ellipse, the major axes of which are inclined to the coordinate axes at a certain angle α. This angle is equal to 45° when the amplitudes of the temperature fluctuations are the same. There exists between Lissajous figures and the phase shift of two sinusoidally varying quantities with the same frequency a simple relation [7]

$$\varphi=2\operatorname{arctg}\frac{M}{N},$$

where M and N are segments intercepted by the x and y axes, inclined at angles of 45° and 135° to the axis $\vartheta(0,\ t)/\vartheta_0$. This relation is valid if the amplitudes of the oscillations of $\vartheta(x,\ t)$ and $\vartheta(0,\ t)$ are the same. In our case, they are different. However, by using a cathode-ray oscillograph and controlling the gain, we can make the amplitudes identical. When this is not possible, another method [7] can be used.

Impulsive action on a substance has recently been employed to determine its thermophysical properties. This action is described mathematically in the form of a delta function or a pulse of finite duration. Here, it is often necessary to know the moment of time at which the maximum temperature is attained at a certain point of the substance. The latter may be determined from the well-known L-transform of the mathematical model of heat transfer to a substance in [8]. The construction of the schemes for identifying the thermophysical properties of substances for impulse methods is similar to the constructions examined here.

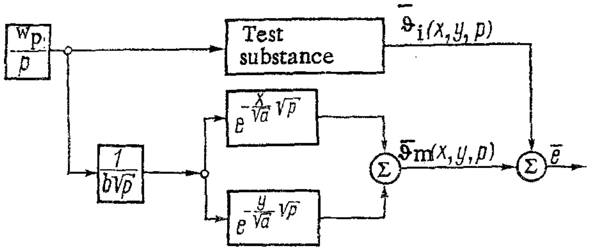

Fig. 8. Identification scheme.

For many substances, the product of the density ρ and the specific heat c has a negligible dependence on temperature but a substantial dependence on thermal conductivity λ. In this case, the function $\lambda(\vartheta)$ must be known. To this end, let us look at an example.

Example. We need to determine (estimate) the temperature dependence of thermal conductivity. We propose to obtain this estimate from the results of control of the constancy of the heat flux $q(0, t) = q_0$ incident on the end surface of a semiinfinite thermally insulated rod and measurements of the temperature $\vartheta(0, t)$ of the end surface.

The temperature dependence of the quantity $\Delta\lambda(\vartheta)$ may be evaluated in accordance with the scheme in Fig. 6. The function $\Delta\lambda(\vartheta)$ is found from the condition of the minimum of the functional

$$E\{\bar{\vartheta}_i(0,\ p), \bar{\vartheta}(0,\ p),\ \Delta\lambda(\vartheta)\} = \left[\bar{\vartheta}_i(0,\ p) - q_0\frac{1}{b_0 p\sqrt{p}} + L\left\{\frac{1}{\lambda_0}\int\limits_0^{\vartheta(0,t)}\Delta\lambda(\vartheta)\,d\vartheta\right\}\right]^2 = e.$$

After the transition to the original, the condition of the minimum acquires the form

$$\frac{2q_0}{b_0\sqrt{\pi}}\sqrt{t} - \vartheta(0,\ t) = \frac{1}{\lambda_0}\int\limits_0^{\vartheta(0,t)}\Delta\lambda(\vartheta)\,d\vartheta\,.$$

In order to use the resulting equation to calculate the function $\Delta\lambda(\vartheta)$, we make note of the fact that $\frac{2q_0}{b_0\sqrt{\pi}}\sqrt{t}$ is calculated; $\vartheta(0, t)$ is measured (thermogram data).

Considering that each moment of time t corresponds to a certain temperature $\vartheta(0, t)$, the difference $\frac{2q_0}{b_0\sqrt{\pi}}\sqrt{t} - \vartheta(0,\ t)$ may be approximated either by a function of temperature

$$\frac{2q_0}{b_0\sqrt{\pi}}\sqrt{t} - \vartheta(0,\ t) = f(\vartheta),$$

or of time

$$\frac{2q_0}{b_0\sqrt{\pi}}\sqrt{t} - \vartheta(0,\ t) = \varphi(t).$$

Then in the first case

$$\Delta\lambda(\vartheta) = \lambda_0\frac{d}{d\vartheta}f(\vartheta)\,,$$

and in the second

$$\Delta\lambda(\vartheta) = \lambda_0\frac{d}{dt}\varphi(t) = \lambda_0\left[\frac{q_0}{b_0\sqrt{\pi}}\frac{1}{\vartheta_t'(0,\ t)} - 1\right].$$

Obviously, the second method of determining $\Delta\lambda(\vartheta)$ is more convenient, since it measures not only $\vartheta(0, t)$, but the derivative $\vartheta_t'(0, t)$. In the first case, the function $\Delta\lambda(\vartheta)$ may be determined by graphical differentiation of the function $f(\vartheta)$.

Of special interest are methods of determining thermophysical properties based on the solution of two-dimensional problems of thermal conductivity, which make it possible to determine thermal conductivity a from measurements of the surface temperature of the test substance. In particular, the identification scheme constructed on the basis of solution of the following problems can be effectively used:

$$\frac{\partial\vartheta(x,\ y,\ t)}{\partial t}=a\left[\frac{\partial^2\vartheta(x,\ y,\ t)}{\partial x^2}+\frac{\partial^2\vartheta(x,\ y,\ t)}{\partial y^2}\right],$$

$$0\leqslant x\leqslant\infty,\ 0\leqslant y\leqslant\infty,$$

$$\vartheta(x,\ y,\ t)=T(x,\ y,\ t)-\Theta,$$

$$\vartheta(x,\ y,\ 0)=0,\ t<0,\ x\geqslant 0,\ y\geqslant 0,$$

$$\vartheta(+0,+0,\ t)=\vartheta_0 u(t),\ \vartheta(x,\ y,\ t)|_{x\to\infty}=\vartheta(x,\ y,\ t)|_{y\to\infty}\to 0,$$

which has the form

$$\vartheta(x,\ y,\ t)=\vartheta_0\,\mathrm{erfc}\frac{x}{2\sqrt{at}}\ \mathrm{erfc}\ \frac{y}{2\sqrt{at}},\ t>0,\ x>0,\ y>0.$$

The source of the temperature $\vartheta(0,\ 0,\ t)$ may be a thin platinum wire placed on the edge of a model of a simulated fourth-space. The measurements of the temperature $\vartheta(l,\ 0,\ t)$ of the surface $y=0$ at the point $x=l$, along with control of the constancy of the temperature ϑ_0, can be used to compute the thermal conductivity a.

Coefficients a and b may also be determined on the basis of the solution of problems of finding the temperature field in the fourth-space when sources of constant power are acting on the limiting surfaces. Such a situation is simpler in terms of practical realization of the boundary conditions. The temperature field may be represented by the sum

$$\vartheta(x,\ y,\ t)=\frac{2\omega_{\text{p}}}{b}\sqrt{t}\left(i\,\mathrm{erfc}\,\frac{x}{2\sqrt{at}}+i\,\mathrm{erfc}\frac{y}{2\sqrt{at}}\right),$$

in which the first term reflects the effect of the sources on the surface $y=0$ and the second term represents the effect of the sources on the surface $x=0$. In the region of the transforms, $\vartheta(x,\ y,\ t)$ has the form

$$\bar\vartheta(x,\ y,\ p)=\frac{\omega_{\text{p}}}{p}\,\frac{1}{b\sqrt{p}}\left(e^{-\frac{x}{\sqrt{a}}\sqrt{p}}+e^{-\frac{y}{\sqrt{a}}\sqrt{p}}\right).$$

This expression may be represented by the structural scheme in Fig. 7. We should note that the solution for the half space

$$\bar\vartheta(x,\ p)=\frac{w_{\text{p}}}{p}\,\frac{1}{b\sqrt{p}}e^{-\frac{x}{\sqrt{a}}\sqrt{p}}$$

does not coincide with the expression for the fourth-space when $y=0$:

$$\bar\vartheta(x,\ 0,\ p)=\frac{w_{\text{p}}}{p}\,\frac{1}{b\sqrt{p}}\left(e^{-\frac{x}{\sqrt{a}}\sqrt{p}}+1\right).$$

In the region of the originals

$$\vartheta(x,\ t)=\frac{2w_{\text{p}}}{b}\sqrt{t}\ i\,\mathrm{erfc}\ \frac{x}{2\sqrt{at}},\ 0\leqslant x\leqslant\infty,$$

$$\vartheta(0,\ t)=\frac{2w_{\text{p}}}{b\sqrt{\pi}}\sqrt{t},$$

$$\vartheta(x,\ 0,\ t)=\frac{2w_{\text{p}}}{b}\sqrt{t}\left(i\,\mathrm{erfc}\,\frac{x}{2\sqrt{at}}+\frac{1}{\sqrt{\pi}}\right),$$

$$\vartheta(0,\ 0,\ t)=\frac{4w_{\text{p}}}{b\sqrt{\pi}}\sqrt{t}.$$

Methods of determining a and b may be realized on the basis of the identification scheme in Fig. 8.

The theoretical formulas were obtained from the condition of minimization of the corresponding functionals.

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HEAT TRANSFER FROM A SPHERICAL SOURCE IN A RAREFIED GAS

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UDC 533.6.011.8

The heat-transfer problem from a sphere in a rarefied gas is solved using a model kinetic equation. Diffuse scattering of gas molecules by the sphere surface with arbitrary energy accomodation is assumed.

A solution of the heat-transfer problem from a sphere in a rarefied gas with arbitrary values of the Knudsen number Kn was published in [1, 2]. Lees [1] used his own four-moment method for solution of the Boltzmann equation. In [2] the BGK model equation was used and complete energy accomodation of gas molecules on the sphere surface was assumed.

In the present study an approximate equation of higher order [3] will be used, ensuring a correct value for the Prandtl number. For the boundary condition we will assume arbitrary molecular energy accomodation on the sphere surface. Such a solution is of interest, first, so that model equations may be compared, second, to evaluate the accuracy of the four-moment method, and third, to provide a quantitative estimate of the energy accomodation coefficient for various gases when theory and experiment are compared.

We will consider a sphere of radius R_0, the temperature of which, T_s, differs from the gas temperature T_∞ in the undisturbed region, while $\tau_s = (T_s - T_\infty)/T_\infty \ll 1$. Then the state of the gas is described by a distribution function close to Maxwellian:

$$f(\mathbf{r},\ \mathbf{v}) = f_\infty [1 + h(\mathbf{r},\ \mathbf{v})],\ \|h\| < 1,$$

$$f_\infty = n_\infty \left(\frac{m}{2\pi k T_\infty}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_\infty}\right), \tag{1}$$

where n_∞ is the numerical gas density in the unperturbed region and k is Boltzmann's constant.

I. M. Gorki Ural State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 3, pp. 428-434, September, 1980. Original article submitted December 7, 1979.

0022-0841/80/3903-0951$07.50